High energy QCD scattering, the shape of gravity on an IR brane, and the Froissart bound

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High-energy scattering in nonconformal gauge theories is investigated using the AdS/conformal field theory (CFT) dual string-gravity theory. It is argued that strong-gravity processes, such as black hole formation, play an important role in the dual dynamics. Further information about this dynamics is found by performing a linearized analysis of gravity for a mass near an infrared brane; this gives the far field approximation to black hole or other strong-gravity effects, and in particular allows us to estimate their shape. From this shape, one can infer a total scattering cross section that grows with center of mass energy as $\ln^2 E$, saturating the Froissart bound.

I. INTRODUCTION

A dominant theme in the past few years of string theory has been the conjectured gauge theory–string theory duality of Maldacena [1]. We are still far from anything approaching a proof of this duality, and we face serious challenges on the most interesting question of using it to infer local properties of string theory (see, e.g., [2]), but considerable evidence has been amassed that one can derive features of gauge theory from bulk gravitational physics. Much of what was initially learned dealt with quantities highly constrained by symmetries, such as the mass spectrum, but more recently attention has been turned to investigating dynamical properties of the theory that are not as tightly circumscribed. This has gone hand in hand with study of spacetimes corresponding to gauge theory vacua with less than the maximal $\mathcal{N}=4$ symmetry; these are typically found by deforming the Lagrangian on the gauge theory side, and correspondingly turning on non-normalizable modes of bulk fields on the string theory side. A particularly interesting recent example is the work of Polchinski and Strassler [3], which addresses a glaring puzzle in the correspondence: How is it that string scattering, which is inherently soft at high energies, reproduces the familiar hard behavior of QCD? By analyzing scattering in the bulk, they find that the soft behavior of string theory conspires with the shape of the bulk wave functions to produce the correct power-law behavior. We can now ask what else we might learn about QCD from the bulk perspective. One obvious question immediately presents itself: What can one say about quantities like the total cross section in very high energy scattering?

Another theme that has recently gained interest is that of the contribution of strong gravitational effects to high energy scattering. For example, in TeV-scale gravity scenarios, based either on large extra dimensions or on strongly warped spacetimes, black holes should be produced once scattering energies pass the fundamental Planck scale near a TeV, and these processes are a very exciting aspect of the phenomenology of these models [4,5] (for a review, see [6]). One can

readily estimate the relevant cross sections based on the observation that at very high energies black hole formation should be well approximated classically (see, e.g., [7]). The naive classical estimate of the cross section to produce a black hole,

$$\sigma \sim \pi r_h^2(E), \tag{1.1}$$

where r_h is the Schwarzschild radius corresponding to center of mass energy E, is a good guide to the magnitude of these effects, as has been confirmed by a more detailed recent analysis of classical high energy scattering [8]. Because of its growth—like $E^{2/D-3}$ in D flat dimensions—this cross section is believed to be a dominant feature of high energy gravitational scattering. Now a second obvious question presents itself: What is the role of strong gravity and in particular black holes in the dual gauge theory?

Indeed, as this paper will argue, our two questions are intimately related. Strong gravitational effects are a dominant feature of the high energy gauge theory scattering, and an estimate analogous to Eq. (1.1), using properties of gravity in deformed AdS backgrounds, yields a high energy cross section of the form

$$\sigma \sim \frac{1}{m^2} \ln^2\left(\frac{E}{E_0}\right), \tag{1.2}$$

where m is the mass of the lightest excitation. This behavior saturates the Froissart bound, which is implied by unitarity and is believed by many to describe the correct high energy behavior of QCD.

A more detailed description of the resulting gauge theory physics remains to be found, but if black hole formation in the bulk is the dominant process at high energy, we might hope to discover the dual physics of Hawking radiation and other interesting effects. At present, however, a significant obstacle appears from questions, raised by Susskind, about whether the relevant gravitational solutions are stable. This is not crucial for them to produce the cross section (1.2), but it is critical in determining the subsequent evolution of the black hole and the corresponding gauge theory dynamics. Regardless, it appears that one may be able to think of the

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gauge theory physics as corresponding to a ''fireball'' that then decays, more or less rapidly depending on these stability questions, into an approximately thermal state.

In outline, this paper will first review the basic setup of a truncated AdS space corresponding to nonconformal gauge theory, along the lines of [3]. This is followed by a discussion of the scales relevant to strong gravitational effects. At moderately high energies one expects to produce black holes small as compared to the anti–de Sitter radius R, but at higher energies these should grow larger than R. Much of the paper deals with the problem of determining their shape. In the full nonlinear theory this is a very difficult problem, but it is argued that a linear analysis can give substantial information about the shape of such solutions. The basic linear equations and boundary conditions are given in Sec. III. A critical role is played by the dynamics that stabilizes the infrared end of the space (the ''radion''), and Sec. IV presents a linear analysis of gravity for both small and large radion masses. In particular, in the heavy radion limit solutions are found that correspond to the far field of an object that could be a black hole. In either limit of the radion mass, one finds cross-section estimates of the form (1.2). Section IV also briefly comments on the relevance of this physics to TeV-scale gravity scenarios in which the extra dimensions may be approximated by a slice of AdS space. Section V contains further discussion of the dual gauge theory physics, and Sec. VI presents conclusions and several open questions. There are two appendixes: one that derives the Green's function for anti–de Sitter space truncated by an infrared boundary—or brane—and another on stabilization mechanisms such as that of Goldberger and Wise [9] and their effective description in geometries whose only boundary is in the infrared.

II. GAUGE THEORY SCATTERING AND TRUNCATED AdS SPACE

On the gauge theory side, we are interested in high energy scattering in a large N $\mathcal{N}=4$ supersymmetric gauge theory with broken conformal symmetry and partially broken supersymmetry. Following Polchinski and Strassler [3], we will assume that on the gravity side, this is dual to a supergravity (or, more precisely, superstring) solution with warped metric

$$ds^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + g_{mn}(y)dy^m dy^n \qquad (2.1)$$

that is approximately of AdS type in a large region, namely,

$$ds^2 \approx \frac{R^2}{z^2}(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu) + R^2 ds_X^2, \qquad (2.2)$$

where X is some appropriate compact manifold. Here R is the AdS radius, and in terms of gauge theory parameters satisfies the approximate relation $R^4 \sim g^2 N \alpha'^2$. In particular, at long distances we can effectively think of the smooth geometry given by Eq. (2.1) as being truncated in the infrared; in some situations it is convenient to think of an ''infrared brane'' as lying at this end of the space. Reference [3] takes the IR end of the space to lie at an arbitrary z, but by an overall rescaling of coordinates $z \rightarrow \lambda z$, $x \rightarrow \lambda x$ we can without loss of

generality take the space to end at $z=R$. This choice also eliminates the need for the different string scales discussed in [3]. Since the scale of the Kaluza-Klein masses is set by $1/R$ (as we will see in more detail), and these are interpreted in the gauge theory as glueballs, this means that the effective QCD scale is $\Lambda_{\text{QCD}} \sim 1/R$.

If we perform scattering in this space, the *conserved* momentum is the gauge theory momentum,

$$p_\mu = -i\frac{\partial}{\partial x^\mu}. \qquad (2.3)$$

As reviewed in [3], we can simply relate glueball scattering amplitudes to bulk scattering as follows. An incoming glueball state has a bulk wave function of the rough form

$$\psi \sim e^{ipx}f(z/R)g(y^i). \qquad (2.4)$$

Here g represents modes of the internal manifold X, and $f(z/R)$ gives the radial wave function in AdS space. The wave functions are exactly of this form in a space that is $\text{AdS}_5 \times \text{S}^5$ truncated by an infrared boundary at $z=R$, but in general the underlying smooth physics and deformation from AdS space will lead to additional mixing. The scattering amplitudes take the form

$$\mathcal{A}_{\text{gauge}}(p) = \int d^{10}x\sqrt{-G}\,\mathcal{A}_{\text{bulk}}(x^\mu, y^i, z)\prod_i \psi_i. \qquad (2.5)$$

Details of the exact form of the wave functions are not needed for our very general purposes, but they generically have large support in the vicinity of the IR boundary. The bulk amplitudes depend on the *proper* momentum as measured by a local observer, which for a given conserved momentum is a function of z:

$$\tilde{p}_\mu(z) = \frac{z}{R}p_\mu. \qquad (2.6)$$

Note that with our conventions $\tilde{p}_\mu = p_\mu$ at the IR boundary.

There are several interesting thresholds that we encounter as we consider scattering at increasing energies. The first is the threshold for scattering in which intermediate string states are important. This threshold is at $p \sim 1/\sqrt{\alpha'}$. At higher energies, the proper energy reaches this scale at a radius given by

$$z_{\text{scatt}} \sim \frac{R}{\sqrt{\alpha'}p}. \qquad (2.7)$$

It is this scattering physics that is important for the amplitudes studied in [3]. However, there are other potentially important effects that [3] does not include.

Specifically, we know that once proper energies pass the 10D Planck scale $M_P \sim g_s^{-1/4}/\sqrt{\alpha'}$, gravity becomes strong and in particular black holes can form. These may initially be best described as highly excited string states, but by the string correspondence principle [10] we expect a smooth crossover from intermediate string states to intermediate

black hole states at the correspondence scale $E_c \sim g_s^{-2}/\sqrt{\alpha'}$. This physics is thus relevant for $p \gtrsim M_P$. If the AdS radius (and other geometrical scales) are much larger than the Planck length, these black holes can be thought of as essentially living in flat space. As we go to higher energies, two things happen. First, one can make black holes at smaller values of z, further into the UV region of the geometry. Any real (as opposed to virtual) black holes of this form would of course be unstable to falling toward the boundary. But, perhaps more importantly, one can make larger black holes. Indeed, for given energy, the radius of a 10D black hole grows as

$$r_h \sim E^{1/7}, \tag{2.8}$$

and consequently the cross section for black hole production at the IR end of the space grows as

$$\sigma \sim E^{2/7}. \tag{2.9}$$

In terms of QCD parameters, $M_P \sim N^{1/4} \Lambda_{QCD}$ and $E_c \sim N^2 \Lambda_{QCD}/(g^2 N)^{7/4}$. Above these energies, one expects possibly interesting consequences of the black hole formation for QCD scattering. One is the power-law growth (2.9). Another comes from the energy dependence of the Hawking temperature of a 10D black hole:

$$T_H \sim E^{-1/7}. \tag{2.10}$$

As is typical in black hole physics, this corresponds to a negative specific heat. Further aspects of this regime have been explored by Dimopoulos, Emparan, and Susskind [11].

As the energy grows further, the black hole size approaches the AdS radius. This occurs at an energy

$$E_R \sim M_P^8 R^7. \tag{2.11}$$

In QCD parameters, this corresponds to $E_R \sim N^2 \Lambda_{QCD}$. At this energy the approximation of these as black holes in background flat space breaks down. The precise nature of black holes at larger energies is an interesting question, but if there are such solutions, we expect them to be completely smeared out over the compact manifold X. Furthermore, effects of the AdS geometry surrounding the IR brane become important. Finding the shape and properties of such a strong gravity region near the IR brane is a nontrivial problem, to which we now turn.

III. LINEARIZED GRAVITY WITH AN IR BRANE

We would like to understand properties of gravity in the vicinity of the IR end of the space (2.1), and at distance scales larger than R. In particular, we would like to determine the nature of possible large black hole solutions at the IR boundary. Of course, this is in general a very complicated problem. For solutions with sizes $\gtrsim R$, we expect any such black hole solutions to be uniformly spread over the compact directions of the internal manifold X, but their shape in the other directions should be nontrivial, and, as we will see, their stability is in question. In order to simplify the problem, we work in an effective description in which we neglect X

and work about a slice of ordinary AdS space terminated in the infrared at $z = R$. Of course, this is just a boundary condition summarizing the much more complicated underlying smooth 10D geometry. The dynamics of this slice can be discussed by introducing an effective tension for an IR brane at $z = R$, as in [12], but that will not be critical to our discussion. Even with these simplifications, at present we lack the tools to find nonlinear gravitational solutions corresponding to black holes on branes in AdS space. However, one can nonetheless find useful information about these solutions from a linearized analysis of gravity, using the following simple principle.

Linearization principle. Suppose that a solution to the full nonlinear field equations exists. In weak-field regions, there must exist a corresponding solution of the linearized equations. One can therefore infer general properties of the nonlinear solution (assuming it exists) by examining solutions of the linearized equations.

Of course, it may be that there is a linearized solution but no corresponding nonlinear solution, or there may be more than one exact solution with the same asymptotic properties. For the present purposes we assume that such violations of existence and uniqueness do not occur.

Using this method, basic structural properties such as the shape and size of the solution can be inferred from the shape of the linearized solutions. For example, if we are studying a black hole, we can infer the approximate location and shape of its horizon by finding the region where the metric perturbation from the background geometry (Minkowski, AdS,...) becomes strong. The corresponding problem of finding black holes on a UV brane was treated by a linear analysis in [13,14], which used this technique to infer the "pancake" shaped black holes of that case. We next turn to a general discussion of the linearized equations, their boundary conditions, brane stabilization, and conditions under which the linearized approximation fails.

A. Linearized gravitational equations

Our starting point is to consider perturbations about the metric (2.2), suppressing X; it is convenient to work using the "height" coordinate y given by

$$z = R e^{y/R}. \tag{3.1}$$

The perturbed AdS metric takes the form

$$ds^2 = (1 + h_{yy})dy^2 + e^{-2y/R}(\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu, \tag{3.2}$$

where we work with the gauge

$$h_{\mu y} = 0. \tag{3.3}$$

This condition does not completely fix the gauge, and in particular allows the gauge transformations

$$y \rightarrow y + \alpha^y,$$
$$x^\mu \rightarrow x^\mu + \alpha^\mu, \tag{3.4}$$

with

$$\partial_\mu \alpha^y + e^{-2y/R} \partial_y \alpha_\mu = 0. \tag{3.5}$$

Here and in the remainder of the paper we adopt the convention that indices on small d-dimensional quantities ($h_{\mu\nu}$, α^μ, etc.) are raised and lowered with the metric $\eta_{\mu\nu}$. The resulting gauge redundancy is parametrized by functions $\alpha^y(x,y)$ and $\beta^\mu(x)$.

There are, furthermore, two possibilities for the allowed α^y. We may fix the region in which we are working, $y \in (-\infty,0)$, and thus demand that gauge transformations not translate the boundary,

$$\alpha^y(0) = 0. \tag{3.6}$$

This gauge, which we refer to as the "straight gauge," allows us to transform h_{yy} to depend only on x. If, on the other hand, we allow gauge transformations that move the boundary,

$$\alpha^y(0) = L(x) \tag{3.7}$$

for some function L, then we may completely eliminate $h_{yy}(x)$. In this gauge, referred to as the "bent gauge," the boundary lies at $y=L(x)$. The four-dimensional fields $h_{yy}(x)$ or $L(x)$ are the present incarnation of the radion field familiar from two-brane scenarios. In either case, the y-independent gauge freedom

$$x^\mu \rightarrow x^\mu + \beta^\mu(x) \tag{3.8}$$

remains.

Einstein's equations for a perturbation with $h_{yy}=0$ were given in [14]; we will find both straight and bent gauges to be useful so we generalize these to $h_{yy} \neq 0$. Letting the source stress tensor be T_{MN}, defining a modified metric perturbation

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} h \eta_{\mu\nu}, \tag{3.9}$$

with $h = \eta^{\mu\nu} h_{\mu\nu}$, and working in $d+1$ dimensions, their perturbations can be shown to be

$$(\partial_d^2 h - \partial^\mu \partial^\nu h_{\mu\nu}) e^{2y/R} - \frac{d-1}{R} \partial_y h - \frac{d(d-1)}{R^2} h_{yy}$$

$$= \frac{1}{M_P^{d-1}} T^y_{\ y}, \tag{3.10}$$

$$\partial_y \partial^\nu (h_{\mu\nu} - h \eta_{\mu\nu}) - \frac{d-1}{R} \partial_\mu h_{yy} = \frac{T^y_{\ \mu}}{M_P^{d-1}}, \tag{3.11}$$

and

$$\Box \bar{h}_{\mu\nu} = \frac{\eta_{\mu\nu}}{2} e^{yd/R} \partial_y (e^{-yd/R} \partial_y h) + e^{2y/R} (-\eta_{\mu\nu} \partial^\lambda \partial^\sigma \bar{h}_{\lambda\sigma}$$

$$+ \partial^\lambda \partial_\mu \bar{h}_{\nu\lambda} + \partial^\lambda \partial_\nu \bar{h}_{\mu\lambda}) + e^{2y/R} (\partial_d^2 h_{yy} \eta_{\mu\nu}$$

$$- \partial_\mu \partial_\nu h_{yy}) + \frac{d-1}{R} e^{yd/R} \partial_y (e^{-yd/R} h_{yy}) \eta_{\mu\nu}$$

$$- \frac{e^{2y/R}}{M_P^{d-1}} T_{\mu\nu} \tag{3.12}$$

from the (yy), (μy), and $(\mu\nu)$ components of Einstein's equations, respectively.

B. Boundary conditions and linear approximation

The linearized Einstein equations (3.10)–(3.12) must be supplemented by boundary conditions. The boundary conditions in the UV ($z=0, y=-\infty$) are simply that the perturbation falls off sufficiently rapidly. Appropriate boundary conditions at the IR brane are the Neumann boundary conditions,

$$n^I \partial_I h_{\mu\nu}|_\partial = 0 \tag{3.13}$$

where n^I is the unit outward normal to the boundary. These can be motivated from orbifold boundary conditions

$$n^I \partial_I h_{\mu\nu}|_- = -n^I \partial_I h_{\mu\nu}|_+ \tag{3.14}$$

about the boundary, or equivalently from the statement that the gravitational field vanishes at the boundary. In the case where there is a source localized on the IR brane,

$$T_{\mu\nu} = S_{\mu\nu}(x) \delta(y), \quad T_{yy} = T_{y\mu} = 0, \tag{3.15}$$

these are modified; integrating Eq. (3.12) over a small neighborhood of the brane gives the Israel matching conditions [15]. Working in the straight gauge, where the boundary is at $y=0$, and combining the result with Eq. (3.14) gives

$$\partial_y (h_{\mu\nu} - \eta_{\mu\nu} h)|_{y=0} - \frac{d-1}{R} h_{yy}(0) \eta_{\mu\nu} = \frac{S_{\mu\nu}(x)}{2 M_P^{d-1}}. \tag{3.16}$$

Alternatively, these may be transformed to the bent gauge using Eq. (3.7). The resulting boundary conditions become

$$\partial_y (h_{\mu\nu} - \eta_{\mu\nu} h)|_{y=L} - 2 \partial_\mu \partial_\nu L + 2 \eta_{\mu\nu} \partial_d^2 L = \frac{S_{\mu\nu}(x)}{2 M_P^{d-1}}. \tag{3.17}$$

If we find solutions to the equations (3.10)–(3.12) with boundary conditions (3.16) or (3.17), we have found a valid approximate solution as long as the metric perturbation remains small,

$$h_{IJ} \ll 1. \tag{3.18}$$

An additional important condition when working in the bent gauge is that the bending remain small [this was used in deriving Eq. (3.17)]:

$$\partial_\mu L \ll 1. \tag{3.19}$$

C. Brane bending and stabilization

If we put a stress tensor of the form (3.15) on the brane, in general it will bend the brane; this phenomenon was studied in [13,14], and we will extend the analysis there to treat the

present situation. For example, we will find that (as expected) a point mass placed on the brane will bend it further into the infrared.

However, in typical examples of nonconformal theories, such as the $N=1^*$ theory [16], the brane cannot be at an arbitrary location, but its position is fixed by the perturbation away from conformality. In [16], this stabilization results from nonvanishing of components of the G flux.[1] This has the effect of giving an effective potential for the location of the brane; in other words, a mass to the ''radion'' which describes shifting the position of the brane. In general this will be summarized in a stress tensor $^{\text{stab}}T_{IJ}$ on the right-hand side of Einstein's equations (3.10)–(3.12). If the radion is very massive, the brane will bend relatively little when the vacuum solution is perturbed.

In the $N=1^*$ theory, deformations of the radion are related to fluctuations of the form $\langle \text{tr } \phi^2 \rangle$. Since the dynamics of [16] are rather complicated, we will instead use a toy model for radion stabilization, along the lines suggested by Csáki *et al.* [17] and in [14], and which can be motivated by studying properties of the Goldberger-Wise model for radion stabilization [9]. Specifically, suppose that we work in the straight gauge, where $h_{yy}=h_{yy}(x)\neq 0$; in this gauge, fluctuations of the radion correspond to fluctuations of h_{yy}. A particularly simple form of the stabilizing stress tensor, compatible with energy-momentum conservation, is

$$^{\text{stab}}T^y_y = \mu^2 M_P^{d-1} h_{yy}(x) e^{yd/R},$$
$$^{\text{stab}}T_{\mu I} = 0, \qquad (3.20)$$

where μ is a constant that is proportional to the mass of the radion. One can think of this as roughly arising from a ''potential'' of the form $U = \mu^2 (h_{yy})^2$, as in [17], or derive it in a limit of the Goldberger-Wise wise model as is discussed in Appendix B. A basic intuition behind the possibility of having a nonzero T_{yy} with other components vanishing is that a very stiff spring can exert a large force with little change in its energy.

We will study two limiting cases of such stabilization: the first $\mu R \ll 1$, in which brane bending is the dominant effect, and the second where $\mu R \gg 1$, and brane bending is effectively eliminated. Although it may be possible to physically achieve either limit in more fundamental models, one expects that commonly the radion mass will take a value $\mu \sim 1/R$, complicating the analysis. In subsequent sections we will see that a particularly important physical question is whether the radion is the lightest massive mode, or whether the first Kaluza-Klein mode of the graviton is lighter.

IV. BLACK HOLES ON IR BOUNDARIES?

We would like to study the properties of strongly coupled gravitational solutions in the vicinity of the IR boundary of the space and specifically search for black holes. The simplest case is a solution with the symmetries of the

d-dimensional Schwarzschild solution, namely, $SO(d-1)$ rotation symmetry. We look for a solution with matter source on the IR boundary or brane. The long range weak field of such a solution should correspond to the long range weak field of a point mass at the boundary. Thus, from the linearization principle, we should be able to detect the presence of a full nonlinear solution and its gross features from studying the linearized equations. Of course, for a given linear solution there could be more than one nonlinear solution, or none; to address the former, we assume that there is a uniqueness result analogous to Birkhoff's theorem; if this is not true, then that has other potentially interesting consequences. And we should also bear in mind that including realistic dynamics for other fields besides the radion and graviton could lead to other richer phenomena.

Therefore, consider a point mass on the boundary,

$$S_{\mu\nu} = 2m \, \delta^{d-1}(x) \delta^0_\mu \delta^0_\nu \qquad (4.1)$$

(the factor of 2 arises from the orbifold treatment of boundary conditions) with a bulk stress tensor that stabilizes the radion, Eq. (3.20), but otherwise vanishes. In the next subsections we will solve for the linearized field of a general source on the brane, in the limits of a stiff or soft radion, and then specialize to the case of such a point mass.

A. The light radion limit

In the case $\mu R \ll 1$ we neglect the stabilizing stress tensor entirely. This problem is most easily solved by passing to the bent gauge, with $h_{yy}=0$. The divergence of the (μy) Einstein equation (3.11) can then be integrated with respect to y to find

$$\partial^\mu \partial^\nu h_{\mu\nu} = \partial^2_d h + f(x) \qquad (4.2)$$

for unknown $f(x)$. Using this in the (yy) equation (3.10) then gives

$$\frac{d-1}{R} \partial_y h = -f(x) e^{2y/R}. \qquad (4.3)$$

This is integrated subject to the boundary conditions (3.17) and those at infinity. Demanding that the field die off at infinity requires that $f \equiv 0$, and therefore that $\partial_y h \equiv 0$. The trace of Eq. (3.17) then determines the position of the boundary,

$$\partial^2_d L = \frac{1}{4(d-1)M_P^{d-1}} S(x). \qquad (4.4)$$

The (μy) equation then implies

$$\partial_y \partial^\nu h_{\mu\nu} = 0, \qquad (4.5)$$

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[1]I thank J. Polchinski for a discussion on this point.

and the gauge freedom (3.8) can then be used to go to transverse gauge,

$$\partial^\nu \bar{h}_{\mu\nu} = 0. \qquad (4.6)$$

Thus the $(\mu\nu)$ equations reduce to

$$\Box h_{\mu\nu} = 0. \qquad (4.7)$$

This is solved using the Neumann Green's function for the scalar Laplacian, defined by

$$\Box \Delta_{d+1}(X,X') = \frac{\delta^{d+1}(X-X')}{\sqrt{-G}},$$

$$\partial_y \Delta_{d+1}(X,X')|_{y=0} = 0 \qquad (4.8)$$

with coordinates $X=(x,y)$ or (x,z). Using the boundary conditions (3.17), the solution is

$$h_{\mu\nu}(X) = -\frac{1}{2M_P^{d-1}} \int d^d x' \sqrt{-g} \Delta_{d+1}(X;0,x')$$

$$\times \left[S_{\mu\nu}(x') - \eta_{\mu\nu} \frac{S^\lambda_\lambda(x')}{d-1} + \frac{\partial_\mu \partial_\nu}{\partial^2} \frac{S^\lambda_\lambda(x')}{d-1} \right]. \qquad (4.9)$$

Specializing to the point mass (4.1), we first solve for the radion from Eq. (4.4), using the d-dimensional Green's function. The result is

$$L = \frac{m}{2(d-1)M_P^{d-1}} \frac{1}{(d-3)\Omega_{d-2} r^{d-3}} \qquad (4.10)$$

where Ω_n is the volume of the unit sphere S^n. The metric can likewise be found using Eq. (4.8). In particular,

$$h_{00} = -\frac{m}{M_P^{d-1}} \frac{d-2}{d-1} \int dt' \Delta_{d+1}(X;0,\vec{0},t') \qquad (4.11)$$

gives the effective scalar potential.

The shape of gravity is thus determined by the scalar Green's function. This is found in Appendix A, and, for a source on the brane, simplifies to

$$\Delta_{p+1}(x,z;x,R) = -\left(\frac{z}{R}\right)^{d/2} \int \frac{d^d p}{(2\pi)^d} \frac{1}{q} \frac{J_{d/2}(qz)}{J_{d/2-1}(qR)} e^{ip(x-x')} \qquad (4.12)$$

with $q^2 = -p^2$.

While the expression (4.9) is difficult to evaluate explicitly, we are particularly interested in the long distance limit where it simplifies. Specifically, assume that $z \ll R$ and/or $x \gg R$. In either case, the integral is then dominated by the region of small qz, and so we can replace the Bessel functions by a small argument expansion. In particular, in the case of the point mass (4.1), we find

$$h_{00}(X) \simeq \frac{mR}{(d-1)M^{d-1}} \left(\frac{z}{R}\right)^d \int \frac{d^{d-1}p}{(2\pi)^{d-1}} \frac{(qR/2)^{d/2-1}}{J_{d/2-1}(qR)}$$

$$\times \frac{d-2}{\Gamma(d/2+1)} e^{i\vec{p}\cdot\vec{x}}. \qquad (4.13)$$

The denominator has poles where qR is one of the zeros $j_{d/2-1,n}$ of the Bessel function $J_{d/2-1}$. Therefore we expect the large r behavior to be dominated by the first pole, and specifically the potential to take the form

$$h_{00} \simeq -\frac{km}{RM_P^{d-1}} \left(\frac{z}{R}\right)^d \frac{e^{-M_1 r}}{r^{d-3}}, \qquad (4.14)$$

where k is a numerical constant and the mass of the lightest Kaluza-Klein mode of the graviton is given by

$$M_1 = j_{d/2-1,1}/R. \qquad (4.15)$$

Note that, as expected, there is no $1/r^{d-3}$ falloff in h_{00}, which would be characteristic of d-dimensional gravity. Without an ultraviolet brane, the effective d-dimensional Planck mass is infinity; in other words, there is no graviton zero mode.

A sufficiently large point mass might be expected to produce a black hole. Specifically, for a given mass, the horizon would be expected to lie in the region where $h_{00} \sim -1$, namely, where g_{00} is expected to vanish, and for sufficiently large m the size of this region would be expected to be larger than R. However, in the present case one cannot explicitly find such a linearized solution. From Eq. (4.10), we find that the condition for the radion field to stay linear, Eq. (3.19), is

$$r \gtrsim \left(\frac{m}{M_P^{d-1}}\right)^{1/(d-2)}. \qquad (4.16)$$

In other words, if we concentrate a mass m in a region smaller than that given by this equation, the brane bending goes nonlinear and forces us to do a nonlinear analysis. One can easily convince oneself that in the present case this happens before $h_{00} \sim -1$. One might extend this analysis by solving the nonlinear generalization of Eq. (4.4) and then find the metric perturbation satisfying Eq. (4.7) in this background, but we leave this for further work. While there are strong gravitational effects at radii less than those given by Eq. (4.16), and while those may well correspond to black hole formation, we cannot presently clearly interpret them as such without treating the physics of the large bending of the boundary.

B. The heavy radion limit

We next solve Einstein's equations (3.10)–(3.12) in the stiff limit $\mu R \gg 1$. With stabilization present, we work in the straight gauge. Taking the divergence of the (μy) equation (3.11) and integrating with respect to y gives

$$\partial^\mu \partial^\nu h_{\mu\nu} - \partial_d^2 h = \frac{d-1}{R} y \partial_d^2 h_{yy} + f(x), \qquad (4.17)$$

again with unknown integration constant $f(x)$. Combining this with the (yy) equation (3.10) then gives

$$\partial_y h = -y e^{2y/R} \partial_d^2 h_{yy} - \frac{Rf}{d-1} e^{2y/R} - \frac{d}{R} h_{yy} - \frac{\mu^2 R}{d-1} h_{yy} e^{yd/R}. \tag{4.18}$$

This must again be solved with boundary conditions (3.16) and the field should fall off sufficiently rapidly at infinity.

In the limit of large μ, the boundary condition (3.16) at $y=0$ can be solved by taking

$$\mu^2 R h_{yy} = \frac{S}{2M_P^{d-1}}, \tag{4.19}$$

which has a solution with $h_{yy} \sim \mathcal{O}(1/\mu^2)$: with a large mass, the deformation of the radion is small. This is precisely as in the cosmological case in [17]. The boundary condition at infinity then implies that $f=0$. In the stiff limit $\mu \to \infty$, we therefore simply set $h_{yy}=0$, and solve the remaining equations using

$$\partial_y h = \frac{e^{dy/R} S}{2(1-d)M_P^{d-1}}. \tag{4.20}$$

Indeed, returning to the (μy) equation, we find that up to a piece that can be gauged away by Eq. (3.8),

$$\partial^\nu \bar{h}_{\mu\nu} = \frac{R}{4d(1-d)M_P^{d-1}} e^{dy/R} \partial_\mu S. \tag{4.21}$$

The remaining ($\mu\nu$) equation then becomes

$$\Box \bar{h}_{\mu\nu} = \frac{R}{2d(1-d)M_P^{d-1}} e^{(d+2)y/R} \left(\partial_\mu \partial_\nu S - \eta_{\mu\nu} \frac{\partial_d^2 S}{2} \right). \tag{4.22}$$

This final equation is most easily solved by using a transverse-traceless projection; specifically, define the transverse projection operator

$$\Pi_{\mu\nu} = \eta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial_d^2}. \tag{4.23}$$

Then the transverse-traceless piece of h is given by

$$h_{\mu\nu}^{\mathrm{TT}} = \Pi_{\mu\lambda} \Pi_{\nu\sigma} \bar{h}^{\lambda\sigma} - \frac{1}{d-1} \Pi_{\mu\nu} \Pi_{\lambda\sigma} \bar{h}^{\lambda\sigma} \tag{4.24}$$

and is easily shown to satisfy

$$\Box h_{\mu\nu}^{\mathrm{TT}} = 0. \tag{4.25}$$

The boundary condition is likewise projected and gives

$$\partial_y h_{\mu\nu}^{\mathrm{TT}} |_0 = \frac{1}{2M_P^{d-1}} \left(S_{\mu\nu} - \frac{1}{d-1} \Pi_{\mu\nu} S \right) \equiv \frac{S_{\mu\nu}^{\mathrm{TT}}}{2M_P^{d-1}}. \tag{4.26}$$

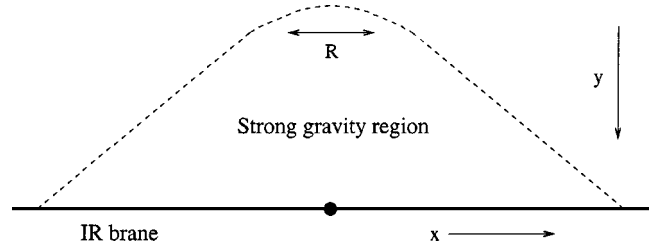These are again solved using the scalar Green's function and give

FIG. 1. Shown is a sketch of the strong-gravity region in the rigid brane limit. This region should be smoothed out in a summit region of size $\Delta x \sim \mathcal{O}(R)$. This curve should give the approximate shape of a black hole horizon.

$$h_{\mu\nu}^{\mathrm{TT}} = -\frac{1}{2M_P^{d-1}} \int d^d x' \sqrt{-g} \Delta_{d+1}(X;0,x') S_{\mu\nu}^{\mathrm{TT}}, \tag{4.27}$$

exactly as in Eq. (4.9). The full metric perturbation is then easily found to be

$$h_{\mu\nu} = h_{\mu\nu}^{\mathrm{TT}} + \frac{\partial_\mu \partial_\nu}{\partial_d^2} h \tag{4.28}$$

with h given by the integral of Eq. (4.20). In particular, the linearized potential of a pointlike static source on the brane is once again given by Eq. (4.11).

We can again enquire regarding the existence of a black hole solution. Asymptotically far from the source, h_{00} is again given by the simple form (4.14). As we approach the source, $h_{00} \approx -1$ at a surface given by

$$M_1 r - yd/R \simeq \ln \left[\frac{km}{RM_P^{d-1} r^{d-3}} \right]. \tag{4.29}$$

This surface is sketched in Fig. 1. We expect that in the exact solution it is rounded off in a region of size $\sim R$ surrounding the summit. Its size grows logarithmically with the mass M. By the linearization principle, this should be the shape of the corresponding black hole horizon. Note that the size of the region, projected onto the infrared boundary, is given by

$$r_h(m) \sim \frac{1}{M_1} \ln \left[\frac{km M_1^{d-3}}{RM_P^{d-1}} \right]. \tag{4.30}$$

As pointed out by Susskind [18], there is a question about the existence of stable black holes with this shape; he has argued, in particular, that any such horizon would thermodynamically prefer to sink into the boundary. To make this more precise, a quick estimate shows that there could be higher entropy solutions with the same conserved energy. This is most easily studied if we imagine that we periodically identify the flat dimensions so that they have finite volume V; for simplicity consider the case of $d=4$. The entropy or area of the plateau at the summit varies like

$$A \sim (e^{-y_{\mathrm{plateau}}/R} R)^3 \sim R^3 \left[\frac{km}{R^2 M_P^3} \right]^{3/4}. \tag{4.31}$$

On the other hand, we might guess that another configuration with the same energy is a black brane, with an energy density $\epsilon \sim m/V$. We can estimate the entropy as that of the usual black three-brane solution (see, e.g., [19]):

$$S \sim \sqrt{N} V^{1/4} m^{3/4}. \qquad (4.32)$$

In the large volume limit, the black brane entropy is clearly larger.

However, note one critical caveat. Our linearized analysis assumed the existence of a stabilization mechanism. For the present discussion of the instability to decay to a black brane to be relevant, there must be a black brane solution to the coupled equations including the stabilizing fields. The question of the existence of such solutions—which corresponds to the question of finding a finite temperature vacuum of the field theory—has been the subject of some discussion; see, e.g., [20]. In some cases it is believed that the field theory supports no corresponding thermally excited state. However, one may check that there is a limit of the Goldberger-Wise mechanism as $m \to 0$ in which the radion mass stays fixed but the back reaction of the Goldberger-Wise field at zero radion displacement vanishes. This suggests that such simple models have black brane solutions like those where there is no stabilization mechanism present. The existence of a stable black brane solution in a general scenario is an interesting and important question that will ultimately determine the fate of the solutions found in this paper, and different theories may yield different results.

C. Intermediate radion mass

We now give a general discussion of the case where the radion mass falls between the two limiting cases. In this case it is more difficult to solve the corresponding coupled equations, but we hope to infer basic properties of the solutions.[2]

In this intermediate case, it may be most advantageous to again work in the bent gauge. We can guess the contribution of the stabilizing stress tensor to the equations of motion in this gauge; in particular, we expect the boundary condition and equations of motion to give

$$\partial_d^2 L + M_L^2 L \sim \frac{S(x)}{M_P^{d-1}}, \qquad (4.33)$$

where $M_L \propto \mu$ is the mass of the radion. Outside a spherical mass distribution with large mass m, we then expect the radion to take the form

$$L \sim \frac{m}{M_P^{d-1}} \frac{e^{-M_L r}}{r^{d-3}}. \qquad (4.34)$$

Now suppose that we take such a mass distribution and gradually compress it; we would like to know what becomes

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[2]Note, however, that by a general argument [see [21] Eq. (62)] it, is always possible to solve for the transverse-traceless part of the metric given the scalar Green's function. Solving for the coupled longitudinal, trace, and radion excitations is more complicated.

important first, large bending, or strong gravity/possible horizon formation, $h_{\mu\nu} \sim \mathcal{O}(1)$. Obviously this depends on the relative magnitudes of the radion mass and the mass of the first Kaluza-Klein (KK) excitation of the graviton. Bending becomes important when the mass is concentrated in a region with size given by

$$\frac{mM_L}{M_P^{d-1}} \frac{e^{-M_L r}}{r^{d-3}} = \mathcal{O}(1). \qquad (4.35)$$

Other strong-gravity effects such as a horizon would be found where

$$\frac{m}{RM_P^{d-1}} \frac{e^{-M_1 r}}{r^{d-3}} = \mathcal{O}(1). \qquad (4.36)$$

For large m, strong gravity becomes important first if $M_L > M_1$, and bending goes nonlinear first if $M_L < M_1$. From the perspective of the dual field theory, this is obvious: the lightest field dominates the dynamics at long distances.

D. Black hole formation in TeV-scale gravity

In the context of TeV-scale gravity, recent work [4,5] has examined black hole formation and decay in high energy collisions. In the most optimistic of scenarios such processes could be visible at the CERN Large Hadron Collider (LHC). Once one pushes to energies higher than the fundamental Planck scale, one makes bigger and bigger black holes, and the properties of these black holes might be used to infer aspects of the geometry of the extra dimensions [4]. One possibility is that the extra dimensions are large and approximately flat, in which case the black hole production cross section at energy E varies as

$$\sigma \sim E^{2/[D(E)-3]}, \qquad (4.37)$$

where $D(E)$ is the number of spacetime dimensions large compared to the Schwarzschild radius of the black hole at the given energy. Another possibility is that, at large enough scales, the extra dimensions in the vicinity of our observable brane take the form of a piece of AdS space, as in the toy model of [12] or the string solutions of [22]. In that case, once one reaches energies large enough to make black holes bigger than the AdS radius scale, the cross section is given by Eq. (4.30), and grows logarithmically:

$$\sigma \sim R^2 \ln^2\left[\frac{ER}{(M_P R)^{d-1}}\right]. \qquad (4.38)$$

Of course, the above comments and questions regarding possible dominance of radion excitations, classical stability of the resulting black holes, etc., would have very important phenomenological consequences. It would be particularly interesting to better understand any instability to black brane formation more deeply. However, closer investigation of these questions likely requires more detailed constructions.

V. HIGH ENERGY QCD SCATTERING AND THE FROISSART BOUND

We now return to investigating aspects of the dual gauge theory description of this physics. Corresponding to different solutions, there are various realizations of dual gauge theories, but there are some generic features. We expect these to include the presence of the four-dimensional field that corresponds to the radion, as well as the tower of KK excitations of the graviton that we think of as glueballs. In addition, there may be other fields. The subsequent discussion applies in cases where these other fields do not significantly alter the dynamics.

Polchinski and Strassler [3] investigated the role of bulk physics in producing partonlike behavior. However, at large energies we do not expect this to be the dominant physics. From the bulk point of view this is clear: at large energies the cross section for strong gravitational effects, such as black holes with sizes given by Eq. (4.30), grows with energy and is expected to be a dominant effect in the physics. Indeed, in QCD it is also known that the total cross section is not dominated by hard processes and grows with energy. Let us compare these more closely.

As described in Sec. IV C, there are two possible explanations for the dominant long distance nonlinear gravitational behavior: brane bending, or strong gravity effects, such as black hole formation, mediated by the first Kaluza-Klein mode of the graviton. Which is dominant depends on the relative magnitude of their masses, μ and M_1. However, in either case, at high center of mass energy, the size of the regions where they become important follows a simple scaling law. Radion dominant scattering processes should set in at

$$r_L \approx \frac{1}{M_L} \ln\left(\frac{M_L^{d-2} E}{M_P^{d-1}}\right) - \frac{d-3}{M_L} \ln \ln\left(\frac{M_L^{d-2} E}{M_P^{d-1}}\right) + \cdots .$$

(5.1)

KK graviton dominant processes should set in at $r \sim r_h$ as given in Eq. (4.30), or equivalently Eq. (5.1) with M_L replaced by M_1. In either case one can estimate the size of the scattering cross section,

$$\sigma \sim \pi r_{L \text{ or } h}^2,$$

(5.2)

which gives[3]

$$\sigma \sim \frac{\pi}{M_L^2} \ln^2\left(\frac{M_L^{d-2} E}{M_P^{d-1}}\right) \quad \text{or} \quad \sigma \sim \frac{\pi}{M_1^2} \ln^2\left(\frac{E M_1^{d-2}}{M_P^{d-1}}\right).$$

(5.3)

In either case we have recovered from bulk physics the $\log^2 E$ behavior associated with the Froissart bound, which is a general bound following from unitarity.[4] To date there has

been no solid argument that QCD saturates this bound at high energies, although this likelihood has been widely discussed.

In hindsight, reproducing this bound is not so surprising as it may seem, and in fact is related to an old heuristic argument for saturation given by Heisenberg in 1952 [25]. He argued that a target hadron is surrounded by a pion field with energy density $\sim e^{-m_\pi r}$, and suggested that inelastic processes will occur when the collision is close enough to locally yield enough energy to create a pion pair. This yields an estimate

$$\sigma \sim \frac{1}{m_\pi^2} \ln^2 \frac{E}{m_\pi^2},$$

(5.4)

similar to the above.

It is amusing to comment that the present discussion suggests that, via the AdS/conformal field theory (CFT) duality, the bulk physics "knows" about boundary unitarity, and that, furthermore, the high energy scattering is approximately described by classical bulk physics.

Note also that, in addition to strong gravitational scattering/black hole formation in the vicinity of the IR boundary, as the energy increases, formation of black holes above the IR boundary becomes possible. However, for a given gauge theory energy, the local energy is largest in the vicinity of the brane, and given that the gravitational cross section rises with energy, it seems plausible that the dominant processes are those localized near the IR boundary.

Of course, we would like to understand more features of the scattering physics. In the radion dominated case, this question depends closely on the dynamics of the radion, whose investigation we leave for future work. In the KK graviton dominated case, the outcome depends intimately on the stability of nonlinear black hole solutions that would correspond to the linearized solutions found in the preceding section. There are two possibilities. One is the classical instability to decay into a black brane. Susskind has suggested that in this case the gravitational solution would sink into the IR boundary and spread out, ultimately forming a black brane at infinitesimal temperature. The corresponding gauge theory dynamics might be described as formation of a fireball that gradually cools as it spreads out and thermalizes. One expects relevant time scales to be given by $t \sim r_h$. Alternatively, if there is no classical instability, one must consider quantum processes. One is tunneling to a black brane, if one exists. If this process is sufficiently slow, or if a corresponding black brane solution does not exist, then plausibly the dominant decay is via Hawking evaporation of the black hole. Note that this is expected to have a time scale parametrically larger in the collision energy.

VI. CONCLUSIONS AND OUTLOOK

Understanding high energy scattering behavior in gauge theories has remained a historically challenging problem. This suggests turning the lens of AdS/CFT duality to focus on it. This paper has argued that a generic phenomenon on the gravitational side, strong gravitational physics such as

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[3]Note that [23] also considered this problem, but came to the different conclusion that the cross section ceases to grow after black holes reach size $\mathcal{O}(R)$.

[4]For a recent review and discussion of the Froissart bound, see [24].

black hole formation, could play a central role in the high energy physics of the gauge theory. Furthermore, black hole formation at high energies is a nearly classical process: for high center of mass energies, horizons can form at weak curvature [8], rendering quantum effects subdominant.

Unfortunately we are not yet able to describe the nonlinear gravitational solutions relevant to this physics, and to do so probably also requires better knowledge of the bulk string theory backgrounds, such as that in [16], in which these processes take place. Nonetheless, this paper has presented, in an "effective theory" approach, an analysis of the linear behavior of the gravitational field. The linear approximation also contains information about its demise, and thus indicates in what regions nonlinear effects are important. In particular, using this approximation, one may estimate high energy scattering cross sections for gauge theory processes, and derive the $\sigma \sim \ln^2 E$ behavior saturating the Froissart bound. This suggests that bulk physics in a sense "knows" about boundary unitarity—although it is not clear what implications, if any, this has for the problem of bulk unitarity.

This picture also suggests a means to investigate the subsequent evolution of the scattered state, although to do so in detail seems to again require more knowledge of the string backgrounds and the nonlinear solutions that live in them, and in particular of the stability properties thereof. In one picture a black hole, once formed, "melts" onto the infrared boundary. In other configurations, there may even be quasistable black holes (only destabilized by Hawking evaporation); it is very interesting to contemplate properties of the dual description of such an object in the gauge theory.

Open questions therefore include more thorough examination of these processes in more detailed models. In particular, one may start with the gravity dual of the $N=1^*$ theory or other analogues. As pointed out, two of the relevant features of the dynamics are the existence of gravity in the bulk, and of a stabilization mechanism for the position of the infrared boundary, that is, the radion field. It would be interesting to better understand the relation between the stabilization of [16] and the Goldberger-Wise mechanism [9], and the question of whether there are more interesting effects beyond those summarized by the effective stabilization stress tensor (3.20). We would like to know what possibilities exist for the radion mass: are there indeed consistent scenarios with the radion mass exceeding the mass of the first Kaluza-Klein mode, so that analogues to black holes can be relevant, and is a large radion mass (as compared to $1/R$) possible? We would like to understand the properties of the resulting nonlinear gravitational solutions and to understand whether they are classically unstable. The latter hinges on the question of the existence of black brane solutions in the presence of the other fields that provide stabilization and other dynamics; the corresponding question in the gauge theory is whether the theory supports a finite temperature phase. It is certainly conceivable that a variety of models result in a variety of different effects.

This paper has probably just scratched the surface of the role of strong gravitational effects in their Maldacena dual gauge theories.

APPENDIX A: SCALAR GREEN'S FUNCTION FOR AdS SPACE WITH AN INFRARED BRANE

This appendix derives the scalar Neumann Green's function in an anti–de Sitter space that has been truncated in the infrared. More general supergravity backgrounds, where the back reactions of other fields are important, will entail more complicated generalizations of this correlator.

The Neumann Green's function is defined to be the solution of the equation

$$\Box \Delta_{d+1}(X,X') = \frac{\delta^{d+1}(X-X')}{\sqrt{-G}}, \tag{A1}$$

where $X=(z,x)$ or (y,x), and the boundary condition is

$$\partial_y \Delta_{d+1}(X,X')|_{y=0} = 0. \tag{A2}$$

This may be solved by the "method of matching," as in [14]. Specifically, the Green's function solves the homogeneous Laplace equation for $z>z'$ and for $z<z'$; these two solutions may then be matched by integrating Eq. (A1) across the singularity at $z=z'$. In order to do so, we work with the Fourier transform

$$\Delta_{d+1}(X,X') = \int \frac{d^d x}{(2\pi)^d} e^{ip(x-x')} \Delta_p(z,z'). \tag{A3}$$

Solutions to Laplace's equation come in the form of superpositions of Bessel functions, $z^{d/2} J_{d/2}(qz)$, $z^{d/2} Y_{d/2}(qz)$, with $q^2=-p^2$. We write the general superposition of such solutions for $z>z'$ and $z<z'$, apply the boundary condition (A2), demand that the Green's function be regular in the UV at $z=0$, and match by integrating Eq. (A1) from $z=z'-\epsilon$ to $z=z'+\epsilon$. The result is straightforwardly found to be

$$\Delta_p = \frac{\pi}{2R^{d-1}} \frac{(zz')^{d/2}}{J_{d/2-1}(qR)} [Y_{d/2}(qz_>)J_{d/2-1}(qR)$$
$$- J_{d/2}(qz_>)Y_{d/2-1}(qR)]J_{d/2}(qz_<). \tag{A4}$$

The Green's function (A3) significantly simplifies with one point on the brane:

$$\Delta_{p+1}(x,z;x,R) = -\left(\frac{z}{R}\right)^{d/2} \int \frac{d^d p}{(2\pi)^d} \frac{1}{q} \frac{J_{d/2}(qz)}{J_{d/2-1}(qR)} e^{ip(x-x')}. \tag{A5}$$

We also need the asymptotics of this Green's function. At either large x or small z, dominant contributions come from

the region with $qz \ll 1$. This means that we can make a small argument expansion in qz to find

$$\Delta_{d+1}(X;0,x') \simeq -\frac{1}{\Gamma(d/2+1)}\left(\frac{z}{R}\right)^{d/2}\int\frac{d^dp}{(2\pi)^d}\frac{1}{q}\left(\frac{qz}{2}\right)^{d/2}$$
$$\times\frac{e^{ip(x-x')}}{J_{d/2-1}(qR)}. \tag{A6}$$

In particular, note that the integrand in Eq. (A6) has no pole at $q=0$—as expected, in this limit the volume is infinite and there is no massless d-dimensional graviton. It does have poles at the masses of each of the Kaluza-Klein modes of the graviton. At long distances the integral is dominated by the mass of the lowest Kaluza-Klein mode, corresponding to the first zero of the Bessel function:

$$M_1 = \frac{j_{d/2-1,1}}{R}. \tag{A7}$$

The static Green's function is obtained by integrating Eq. (A3) over time. Its asymptotic behavior is given in terms of this mass:

$$\int dt'\Delta_{d+1}(X;0,\vec{0},t') \simeq \hat{k}\left(\frac{z}{R}\right)^d\frac{e^{-M_1 r}}{Rr^{d-3}}, \tag{A8}$$

where $\hat{k}$ is a numerical constant.

APPENDIX B: STABILIZATION, GOLDBERGER-WISE OR OTHERWISE

In this appendix we outline the basics of the Goldberger-Wise mechanism [9], with particular emphasis on large (in the limit, infinite) brane separation, and relate it to the stabilization stress tensor (3.20) used in the text.

We begin by thinking of the situation with AdS space truncated by a brane in the IR and one in the UV. The basic idea of the Goldberger-Wise mechanism is to postulate the existence of a massive bulk field, with Lagrangian

$$S[\phi] = -\frac{1}{2}\int d^{d+1}X\sqrt{-G}[(\nabla\phi)^2+m^2\phi^2], \tag{B1}$$

and such that the *value* of the field is fixed on both the IR and UV branes.[5] Bringing the branes either too close or too far raises the value of the action $S[\bar{\phi}]$ at the corresponding static solution $\bar{\phi}$ of the equations of motion. This therefore generates a potential for the radion.

Note that the Goldberger and Wise original analysis assumed $m^2 > 0$, but the mechanism works for $m^2 < 0$ [21]. This is presumably related to the stabilization evident in Polchinski and Strassler's work, which involves a CFT perturbation that is relevant, and hence has dimension $\Delta < 4$,

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[5]Goldberger and Wise actually considered more generally a potential for the values of ϕ on the boundary, but we omit this unneeded generalization.

corresponding to $m^2 < 0$. While we expect the dynamics to be qualitatively similar, it would be interesting to further elucidate the precise relationship between the two schemes.

We are interested in applying the mechanism to the limiting case where the UV brane is moved to infinity. We therefore generalize the boundary condition at this end of the space to state that, as $y \to -\infty$,

$$\phi \to \phi_0(e^{k_-y}+Ae^{k_+y}) \tag{B2}$$

for fixed ϕ_0 and arbitrary A. Here the exponents are the familiar quantities

$$k_{\pm} = \frac{d}{2R}\pm\sqrt{d^2/4R^2+m^2}. \tag{B3}$$

The boundary condition at $y=L$ is then taken to be $\phi(L) = \phi_L$, fixing A.

If we compute the action (B1) as a function of L, we find the potential

$$V = \frac{1}{2}\int dy\, e^{-dy/R}[(\partial_y\bar{\phi})^2+m^2\bar{\phi}^2]$$
$$= \frac{\phi_0^2}{2}\left[\frac{d}{R}e^{(k_--k_+)L}-2k_+\frac{\phi_L}{\phi_0}e^{-k_+L}\right.$$
$$\left.+k_+\left(\frac{\phi_L}{\phi_0}\right)^2e^{-(k_++k_-)L}+C\right], \tag{B4}$$

where C is a cutoff-dependent but L-independent term. This potential has extrema at

$$e^{k_-L} = \frac{\phi_L}{\phi_0}\frac{k_+^2\pm\sqrt{k_+k_-(k_+k_--k_+^2+k_-^2)}}{k_+^2-k_-^2}. \tag{B5}$$

The radion mass, given by $V''(L)$, grows with the mass m of the scalar field; thus the limit of large scalar mass is one way of motivating the limit of large radion mass used in Sec. IV B.

Note that we can also verify the approximate form of the stress tensor used in Eq. (3.20). From Eq. (B1) we find

$$T_y^y \propto (\partial_y\bar{\phi})^2-m^2\bar{\phi}^2. \tag{B6}$$

The $L=0$ vacuum value of this is treated as part of the background solution. The stabilizing stress arises from the variation of this as L, thus $\bar{\phi}$, varies. In particular, we find

$$\frac{\partial T_y^y}{\partial L} \propto \frac{\partial A}{\partial L}[-4m^2+2A_0(k_+^2-m^2)e^{(k_+-k_-)y}]e^{yd/R}. \tag{B7}$$

The first term has the expected form of Eq. (3.20). The second term is a correction that has support near the lower boundary at $y=0$, but otherwise is small, and vanishes in the limit $m \to \infty$. It also may be checked that there is a limit $m \to \infty$, $\phi_0 \to 0$, such that the radion mass stays fixed (or goes to infinity), but the vacuum back reaction of $T_{IJ}(L=0)$ on the metric stays small.

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